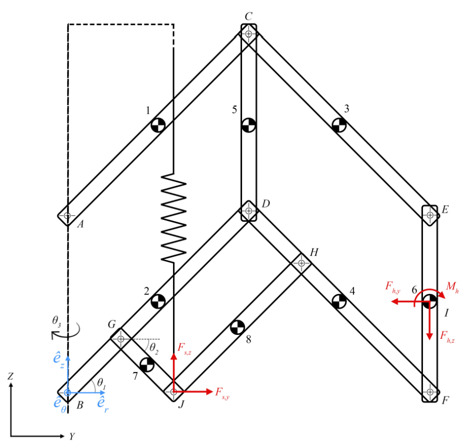
Spring Loaded Double Pantograph : Static Analysis

The static analysis provides a system of equations that can be solved to determine the internal forces in the SLDP mechanism for any given configuration and external load. Figure 1 illustrates the external forces acting at joints I and J. Due to the double-pantograph properties, links 7, 4, and 3 are in parallel, as are links 2, 8, and 1.



Nomenclature	
g	gravity vector
K	spring stiffness parameter
m	counterweight mass
M_h	joint reaction moment magnitude
N	Total number of mechanism Degrees of Freedom
ω	angular velocity
F_h, z	human force in $\hat{e_z}$ direction
F_h, y	human force in $\hat{e_r}$ direction
F_s, z	spring force in $\hat{e_z}$ direction
F_s, y	spring force in $\hat{e_r}$ direction
L_2	link 2 length
L_4	link 4 length
L_6	link 6 length
θ_1	angle from horizontal to link 2
θ_2	angle from horizontal to link 7
θ_3	angle from Y axis to plane of mechanism

Figure 1: Definition of coordinate systems and kinematic parameters with accompanying nomenclature

1 Analysis of Link 2

We begin our analysis with link 2, as shown in Figure 2a. Applying force and moment balance yields:

$$F_{D2,y} + F_{G2,y} - F_{B2,y} = 0 \tag{1}$$

$$F_{D2,z} + F_{G2,z} - F_{B2,z} = 0 \tag{2}$$

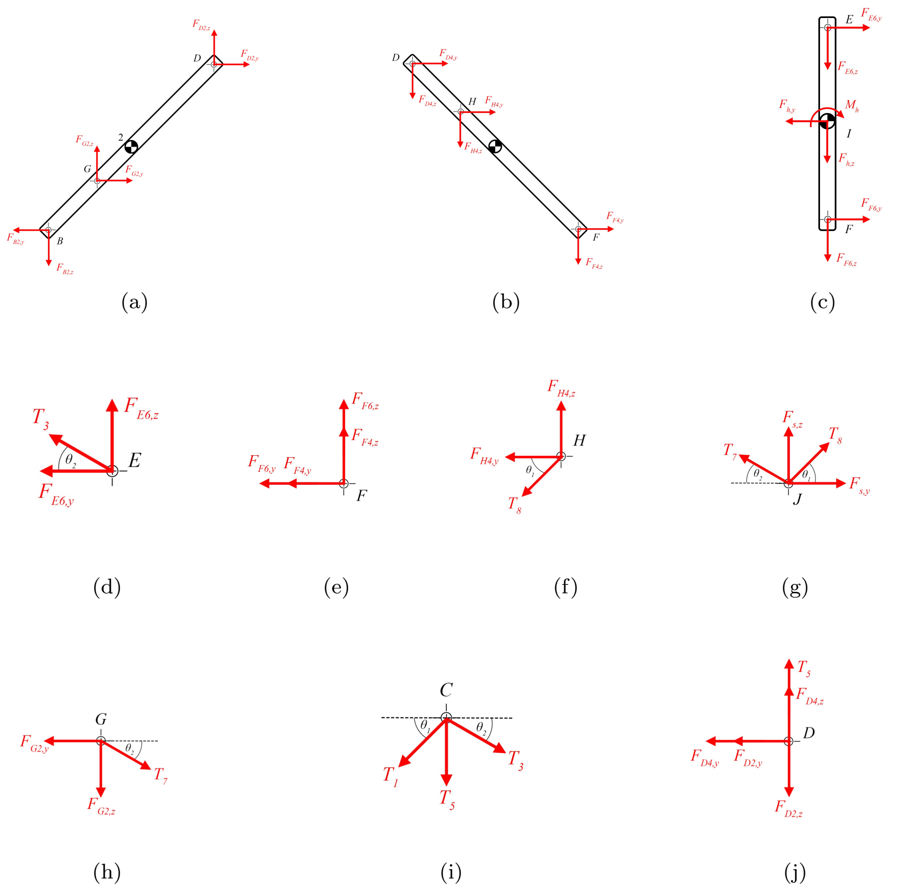


Figure 2: Forces on various links and joints

$$\begin{aligned} L_{GD}\sin\theta_1 F_{G2,y} - L_{GD}\cos\theta_1 F_{G2,z} \\ -(L_{BG}+L_{GD})\sin\theta_1 F_{B2,y} \\ +(L_{BG}+L_{GD})\cos\theta_1 F_{B2,z} = 0 \end{aligned} \tag{3}$$

where $F_{xy,z}$ represents the force at point x in the y or z direction, and L_{xy} is the length between points x and y.

2 Analysis of Link 4

For link 4, as illustrated in Figure 2b, we have:

$$F_{F4,z} + F_{H4,z} + F_{D4,z} = 0 \tag{4}$$

$$F_{F4,y} + F_{H4,y} + F_{D4,y} = 0 \tag{5}$$

$$\begin{aligned} -L_{HF} \sin\theta_2 F_{H4,y} + L_{HF} \cos\theta_2 F_{H4,z} \\ -(L_{DH} + L_{HF}) \sin\theta_2 F_{D4,y} \\ +(L_{DH} + L_{HF}) \cos\theta_2 F_{D4,z} = 0 \end{aligned} \tag{6}$$

3 Analysis of Link 6

For link 6, which connects to the end effector, as illustrated in Figure 2c, we have:

$$F_{F6,z} + F_{H,z} + F_{E6,z} = 0 \tag{7}$$

$$F_{F6,y} - F_{H,y} + F_{E6,y} = 0 \tag{8}$$

$$F_{F6,y} - F_{E6,y} = \frac{2}{L_{EF}} M_H \tag{9}$$

where M_H is the moment applied at point H.

4 Analysis of Joints

Links 1, 3, 5, 7, and 8 can be modeled as two-force members with a constant axial force (denoted T_x). For joint E:

$$F_{E6,z} + T_3 \sin\theta_2 = 0 \tag{10}$$

$$F_{E6,y} + T_3 \cos\theta_2 = 0 \tag{11}$$

For joint F:

$$F_{F4,z} + F_{F6,z} = 0 \tag{12}$$

$$F_{F4,y} + F_{F6,y} = 0 \tag{13}$$

For joint H:

$$F_{H4,z} - T_8 \sin\theta_1 = 0 \tag{14}$$

$$F_{H4,y} + T_8 \cos\theta_1 = 0 \tag{15}$$

For joint J, where the spring force F_s is applied:

$$F_{s,z} + T_7 \sin\theta_2 + T_8 \sin\theta_1 = 0 \tag{16}$$

$$-T_7 \cos\theta_2 + T_8 \cos\theta_1 + F_{s,y} = 0 \tag{17}$$

For joint G:

$$F_{G2,z} + T_7 \sin\theta_2 = 0 \tag{18}$$

$$-F_{G2,y} + T_7 \cos\theta_2 = 0 \tag{19}$$

For joint C:

$$T_5 + T_1 \sin\theta_1 + T_3 \sin\theta_2 = 0 \tag{20}$$

$$-T_1 \cos\theta_1 + T_3 \cos\theta_2 = 0 \tag{21}$$

For joint D:

$$T_5 + F_{D4,z} - F_{D2,z} = 0 \tag{22}$$

$$F_{D4,y} + F_{D2,y} = 0 \tag{23}$$

The analysis presented here assumes static equilibrium. For dynamic situations, these equations would need to be augmented with inertial terms to account for accelerations of the links and the payload. Furthermore, the spring force F_s would be a function of the mechanism's configuration, typically modeled as a linear spring force:

$$F_s = k(\Delta l) \tag{24}$$

where k is the spring constant and Δl is the change in spring length from its rest position.